Variable Annuities
and
Policyholder Behaviour

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Risk Day, 11.9.2015

Aim

- To understand what a Variable Annuity is,
- To understand the different product features and how they interact,
- To understand the risk management of Variable Annuities and their hedging,
- To understand the consequences of policyholder behaviour on valuation and hedging.

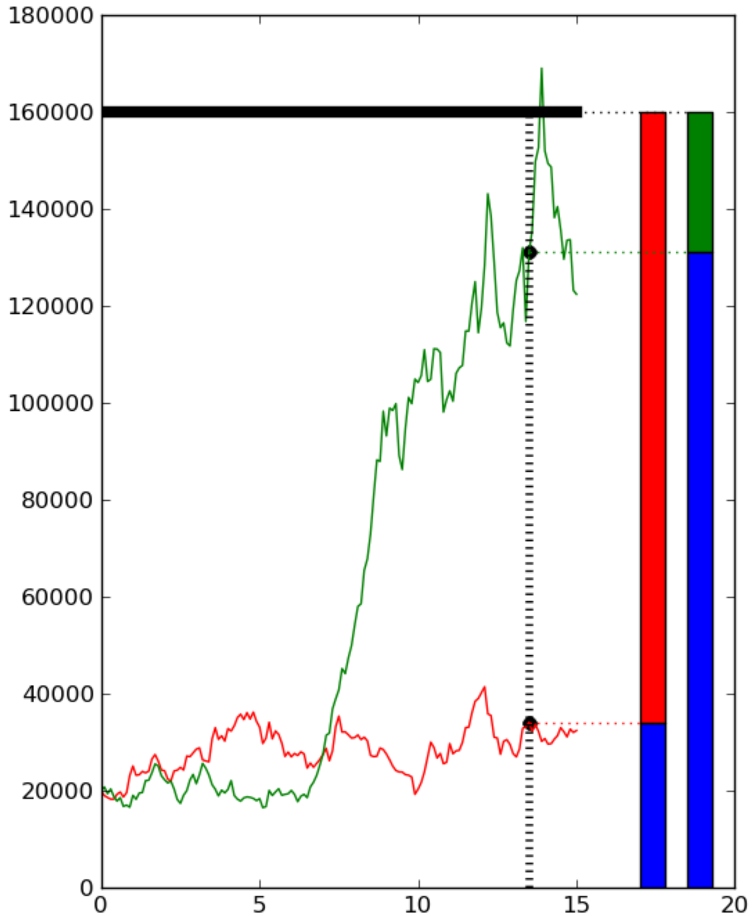
What is a variable annuity?

A Variable Annuity is a Fund(s) plus additional Insurance Benefits.

- Insurance Benefits can be of different forms:
  - In case of Death $\rightsquigarrow$ Guaranteed Minimum Death Benefit (GMDB),
  - For saving $\rightsquigarrow$ Guaranteed Minimum Accumulation Benefit (GMAB), and
  - In case of regular income (annuity) $\rightsquigarrow$ Guaranteed Minimum Withdrawal Benefit (for Life) (GMWB/GLWB)
- The product has tax benefits.

GMDB

GMDB Example



- GMDB: 160'000, initial fund value: 20'000, policy term 25 years.
- We consider two possible outcomes (*trajectories) and assume that the person dies after 17 years. Then we have the following:*

	"green"	*"red"*
Fund Value	*130'000*	*35'000*
Death Benefit	*160'000*	*160'000*
Loss Insurer	*30'000*	*125'000*

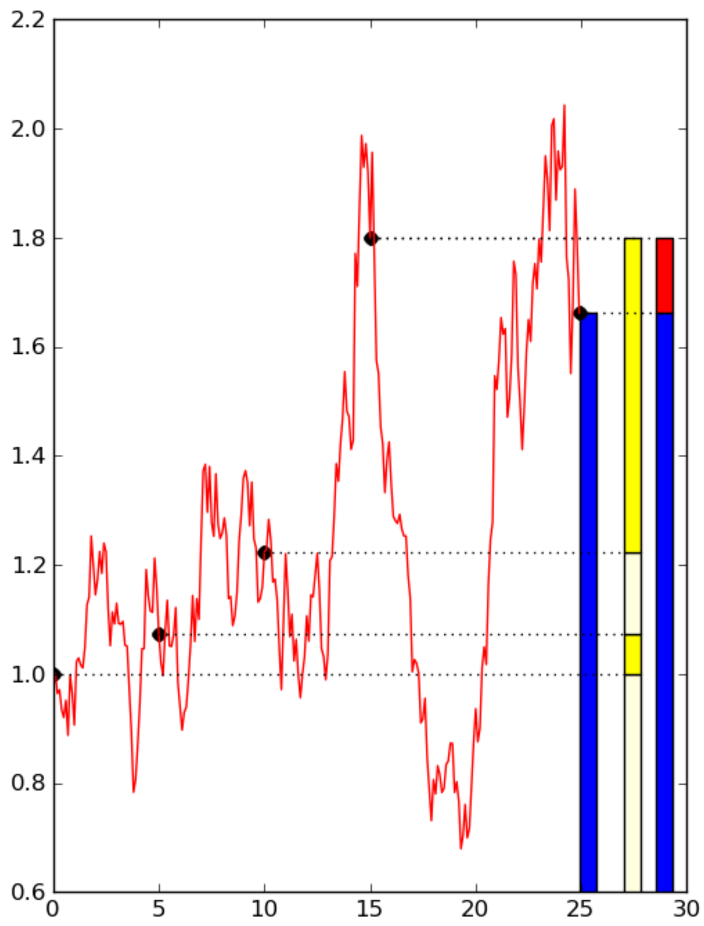
- *Hence the insurer needs to be able to sell the underlying fund at 160'000 if its value is below this amount, independently of its value. This is called a put option with strike 160'000.*
- *In the good outcome ("green"), this guarantee has a value of 30'000. In the bad outcome ("red"), the guarantee has a value of 125'000.*

Insurance Protection within variable annuities

Variable Annuity	Insurance Protection
GMDB (Death Benefit)	Protection in case of death
GMAB (Accumulation Benefit)	Policyholder survives a certain time
GMIB (Income Benefit)	Policyholder survives a certain time, regular income
GMWB (Withdrawal Benefit)	Temporary Annuity (potentially deferred)
GLWB (Life Benefit)	(Deferred) Annuity, longevity

GMAB - Ratchet 5 years

GMAB Ratchet Example 5y

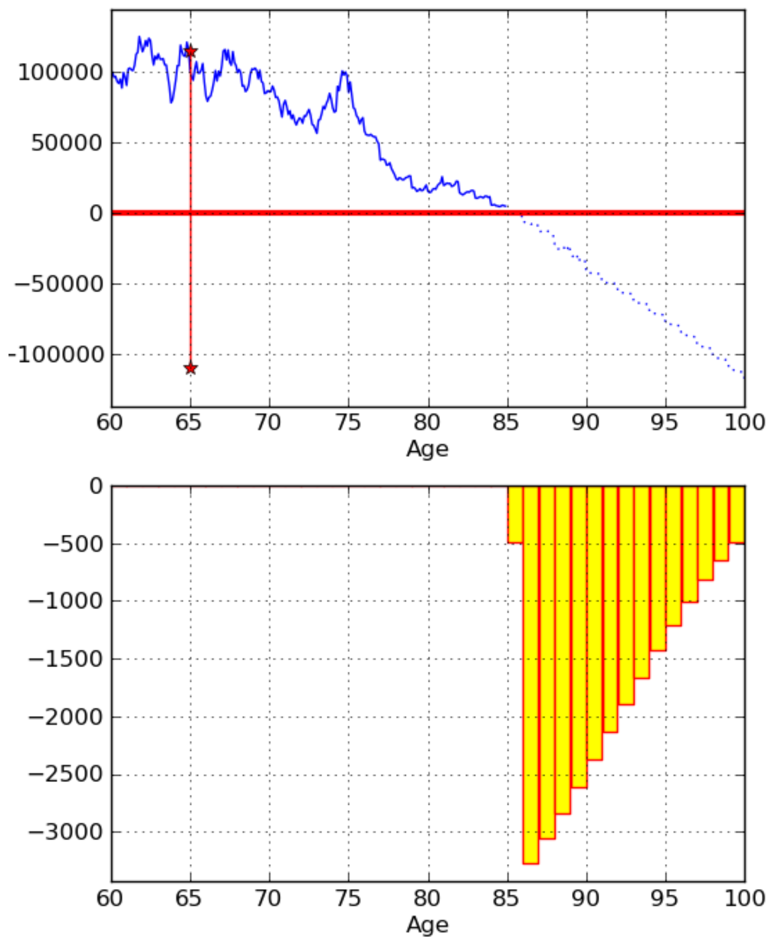


- We assume an initial fund value of 1M.
- This rider to the insurance policy ensures that the policyholder receives the maximum value of the fund attained in the past.
- This maximum is evaluated at regular time intervals.
- In case of a 5 yr ratchet, we have the high water marks as follows:

Time	Fund Value	Ratchet
0	1.00	1.00
2	1.00	1.14
5	1.07	1.14
7	1.07	1.25
...		
20	1.79	1.79
24	**1.79**	**1.92**

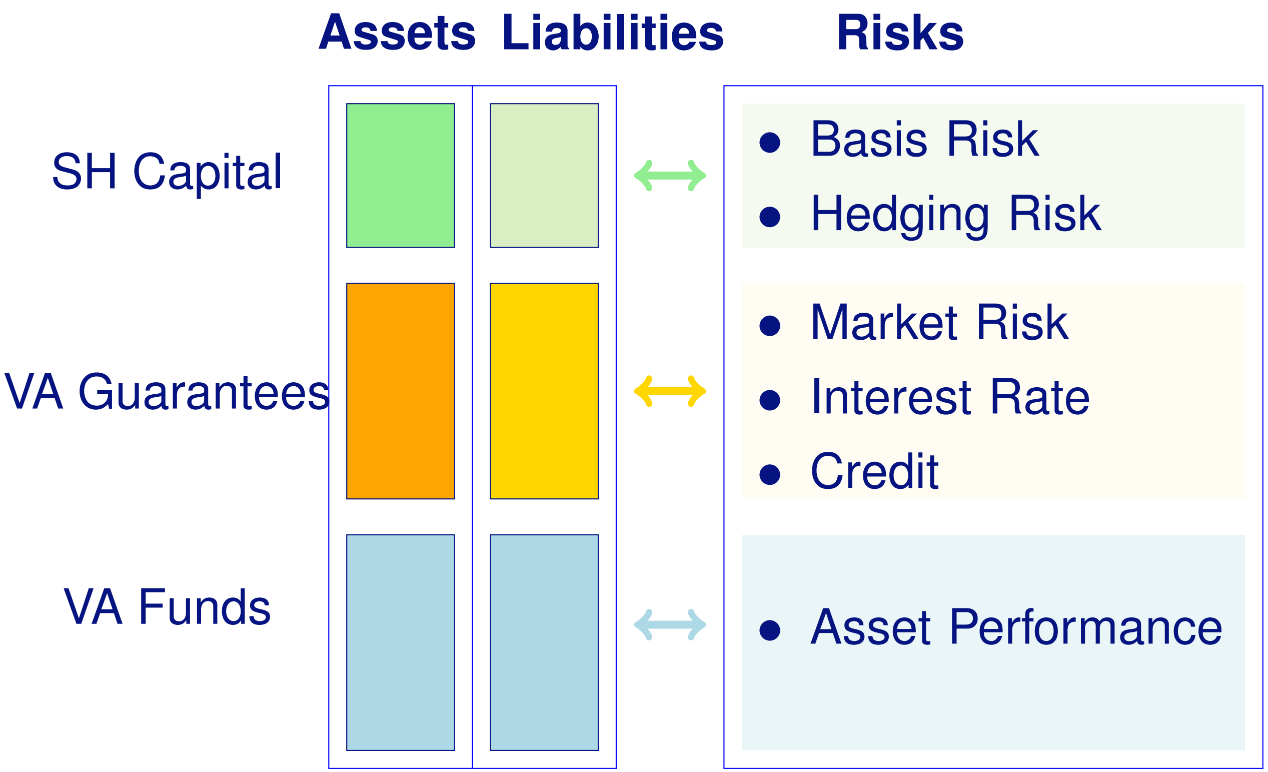
Example GMWB

GLWB Example



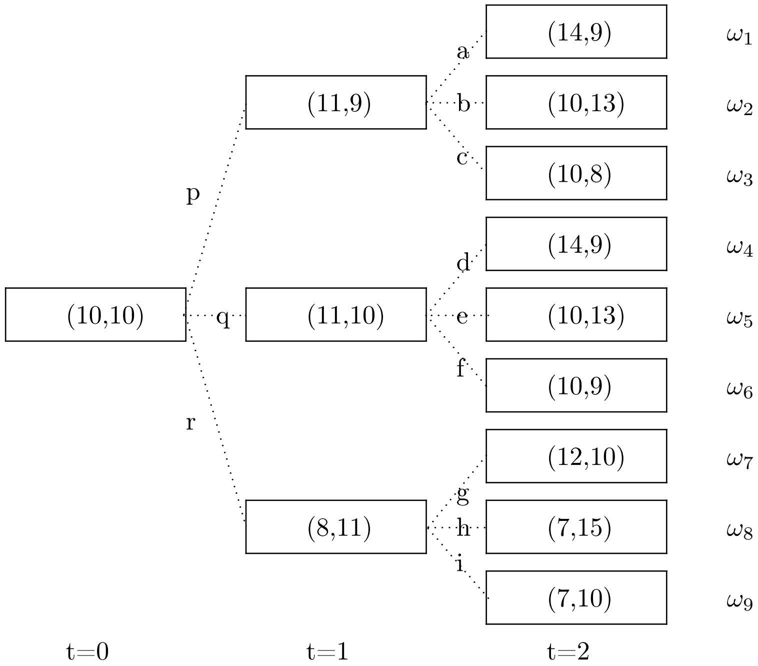
- The Policyholder has invested 100,000 at age 60 and bought a GLWB (Guaranteed Minimum Withdrawal Benefit for Life), with the right to withdraw 5% pa.
- The benefit base GWB has increased if $GWB = c116'000$ because he did not withdraw before age 65, so can now withdraw up to c5’800 pa.
- The fund is depleted at age 85 and the guarantee kicks in.
- The expected guarantee (“yellow”) reduces as more and more policyholders die.

Balance Sheet



Methods to value variable annuities

Valuation Tree



- Explicit formula or recursion (only insurance valuation and very simple variable annuities),
- Solution of Black-Scholes-Merton differential equation (different methods including tree method),
- Monte Carlo Simulation (this is the approach most often used for variable annuities).

Monte Carlo is most commonly used for VA since it is very versatile and can also cope with very complex option structures, such as ratchets.

Steps in Valuation of VA

Determine the number of people which benefit: Since only a tiny percentage of the whole inforce dies within a given year, one only needs to provide the respective GMDB cover to them. Similarly the GMAB cover is paid only to the people surviving the entire term of the policy. Hence we need to determine the respective percentages. This is done by means of life decrement tables.

Calculate what these people receive: Then we need to know what the respective policyholders are entitled to. Assume, for example, the people dying aged 40. They are entitled to get a GMDB at a certain level. Hence we need to determine the number of the corresponding units of guarantees. For our 40 year old person this would be put options at a strike price.

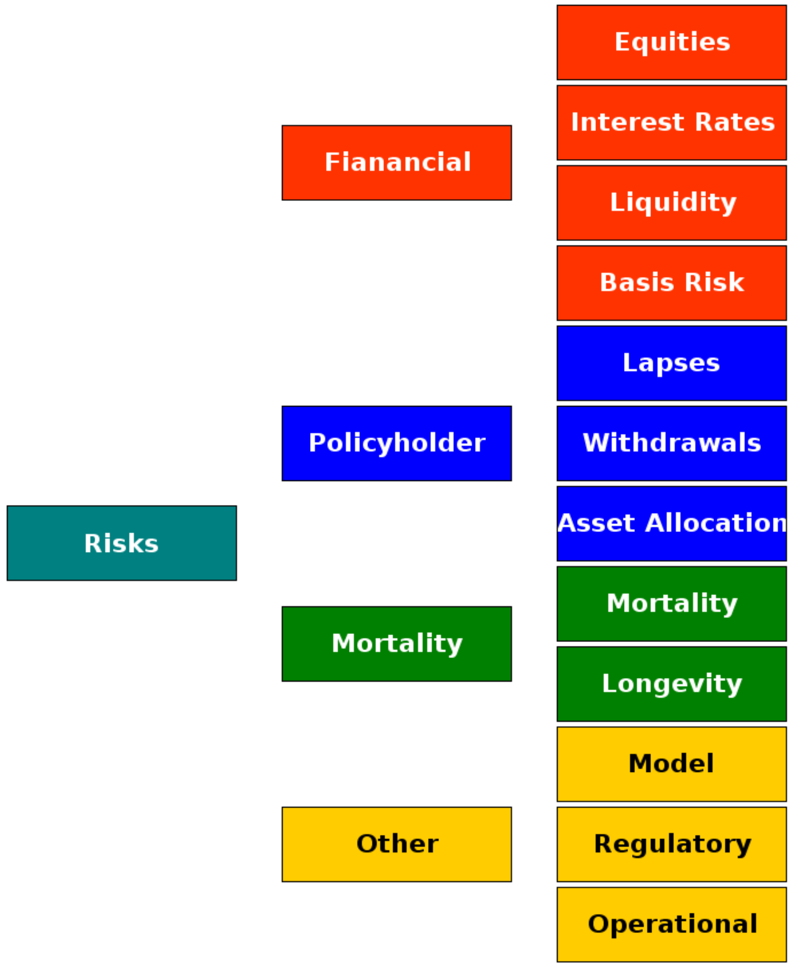
Calculate the value: At this point we know the valuation portfolio of guarantees representing the VA guarantee (eg number of instruments and their characteristics). We now need to value them. For our example this is done via the Black-Scholes formula.

Value of Guarantee – Trading Grid

Equity Level	π	δ	γ	ρ	ν
-50 %	65777.63	-33818.56	39673.78	-2341692.30	86714.87
-40 %	59397.95	-36051.50	42961.58	-2241519.81	111455.01
-30 %	53734.40	-37319.40	44743.99	-2136639.66	133559.18
-20 %	48710.53	-37826.27	45431.79	-2030460.53	152596.75
-10 %	44254.99	-37744.13	45168.28	-1925345.90	168470.76
-5 %	42219.51	-37531.90	44705.65	-1873714.26	175247.49
0 %	40301.71	-37230.02	44052.73	-1822905.27	181284.49
+5 %	38494.06	-36854.69	43243.46	-1773034.67	186616.49
+10 %	36789.41	-36419.96	42315.00	-1724192.41	191283.08
+20 %	33662.33	-35417.69	40235.70	-1629845.32	198789.30
+30 %	30871.11	-34295.33	38026.66	-1540199.53	204141.09
+40 %	28372.90	-33102.03	35808.25	-1455379.92	207655.90
+50 %	26130.91	-31872.16	33640.82	-1375369.59	209618.73

VA risks

Risk Landscape



Financial Risks relate to capital markets, such as equity risks, but also the ability to trade at a certain point in time.

Policyholder Behaviour Risks relate to the behaviour of the policyholder at a given point in time.

Insurance Risks relate to the pure demographic risks such as mortality.

Other Risks summarise the remainder of risks such as a rogue trader, etc.

Short Term vs Longer Term Risks

Short Term

- Equity price,
- Interest rates,
- Operational risk / key man risk,
- Lapses,
- Liquidity,
- Basis Risk.

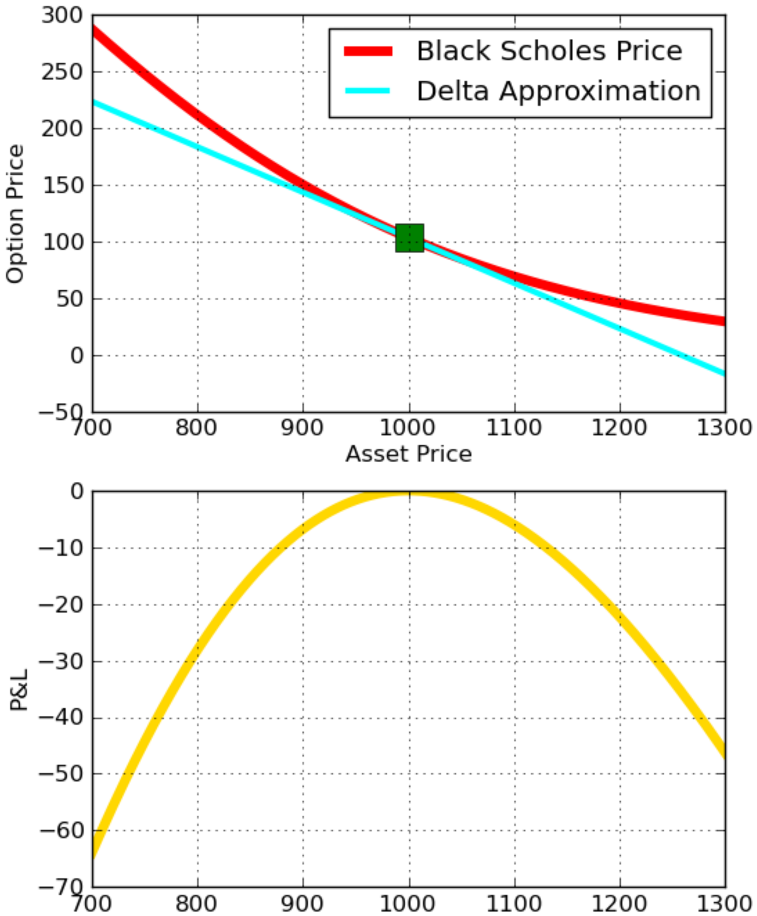
Longer Term

- Longevity,
- Long term volatility,
- Interest rates,
- Policyholder behaviour (lapses, . . .).

Need to monitor short term risk closely and continuously. Regular MI and respective risk appetite statements are necessary.

Aim of Hedging

Approximation



The lower figure shows the P&L (in yellow) for a given hedging strategy (upper figure) with the aim to approximate the value of a VA, which depends among other things on:

- Equity prices, equity volatility,
- Interest rates,
- Mortality,
- Lapses, utilisation ...

Aim of hedging: immunising balance sheet of insurance company.

Hedging Strategy

A hedging strategy needs to consider and establish objectives in respect of:

Economic Risks: Which economic risks are hedged and to what extent?

Financial Statement Risks: How important are the risks regarding the publicly stated accounts and to what extent is there a need to hedge them?

Regulatory Capital Risks: What are the regulatory capital risks and to what extent need they be hedged?

A hedging strategy needs to establish objectives with respect to the different dimensions and define a corresponding risk appetite.

Hedge Strategies

$$\Delta f(S,t,r,\sigma) = \underbrace{\frac{\partial f}{\partial S}}_{Delta} \Delta S + \frac{1}{2} \underbrace{\frac{\partial^2 f}{\partial S^2}}_{Gamma} (\Delta S)^2 + \underbrace{\frac{\partial f}{\partial t}}_{Theta} \Delta t$$
$$+ \underbrace{\frac{\partial f}{\partial r}}_{Rho} \Delta r + \underbrace{\frac{\partial f}{\partial \sigma}}_{Vega} \Delta \sigma + \dots$$

Trivial Hedge: Nothing is hedged and the insurance company keeps entire risk.

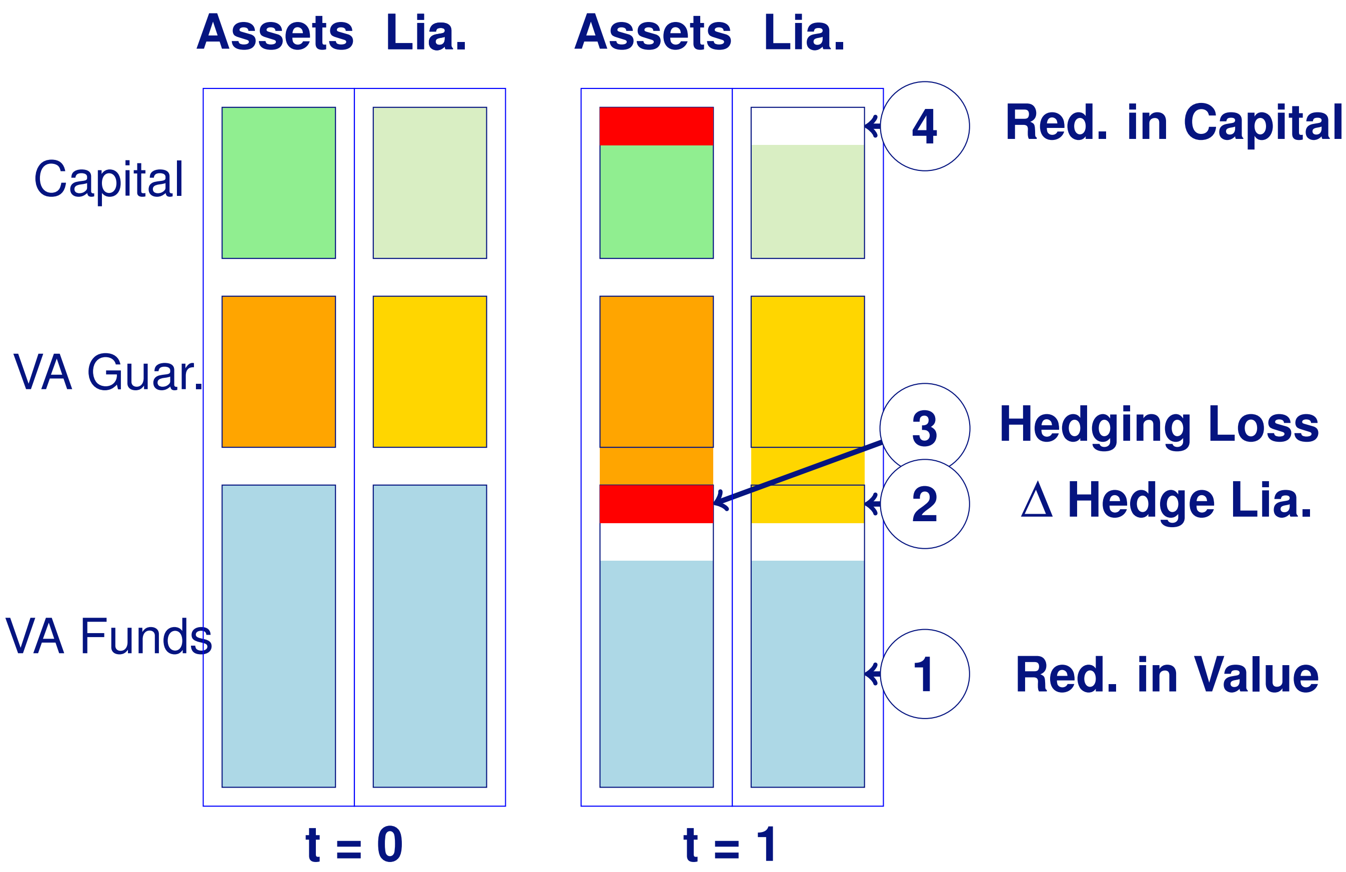
δ-hedge: Only the equity part is hedged, no interest rate hedge. A δ–γ–hedge is a variant of this, where equities are hedged more accurately than with a pure δ–hedge.

δ–ρ–hedge: Interest rates and equities are hedged.

3 greeks hedge: δ, ρ and the equity volatility ν is hedged.

Macro Hedge: The aim is to hedge the tail (or big movement) risks, potentially however trading-off protection against the accuracy of the hedge for smaller magitude movemens.

Balance Sheet



Risk Management for VA in general

- The best way to reduce VA risk is normally to have outperformance in the underlying investment fund. Hence the choice and monitoring of fund performance is very important.
- The only way to really mitigate the VA risks completely is to sell them to a third party, otherwise there are always remaining residual risks.

Risk management for VA is vital.

Policyholder behaviour

Change Asset Allocation: The policyholder can change his asset allocation and invest in different assets, which are more or less risky.

Top up investment: The policyholder can invest an additional amount in the underlying fund. This can change the guarantees.

Lapse: The policyholder can end the policy.

Start withdrawing: The policyholder can start to withdraw money from the fund.

Change amount of withdrawal: Within a given period, the policyholder can decide to withdraw more or less.

Partial Surrender: Withdrawing more than regularly allowed.

Sell Policy: He can sell the policy to a third party to monetize the value of the policy.

Policyholder behaviour is a risk that needs to be considered, in particular for the product design. One needs to avoid product designs, that promote cristallisation of losses for many policyholders at the same time.

Value of Guarantee at Inception

	Instrument	Strike	Amount %age	**Value**
0	Put Fund	100000	0.1 %	8.7
1	Put Fund	107177	0.2 %	17.7
2	Put Fund	114870	0.2 %	27.7
3	Put Fund	123114	0.2 %	39.5
4	Put Fund	131951	0.2 %	53.6
5	Put Fund	141421	0.2 %	70.6
...				
9	Put Fund	186607	0.3 %	146.0
10	Put Fund	200000	0.4 %	181.8
...				
25	Put Fund	200000	86.6 %	34789.4
	Total			**40311.7**

Impact of Lapse on Hedge Liability

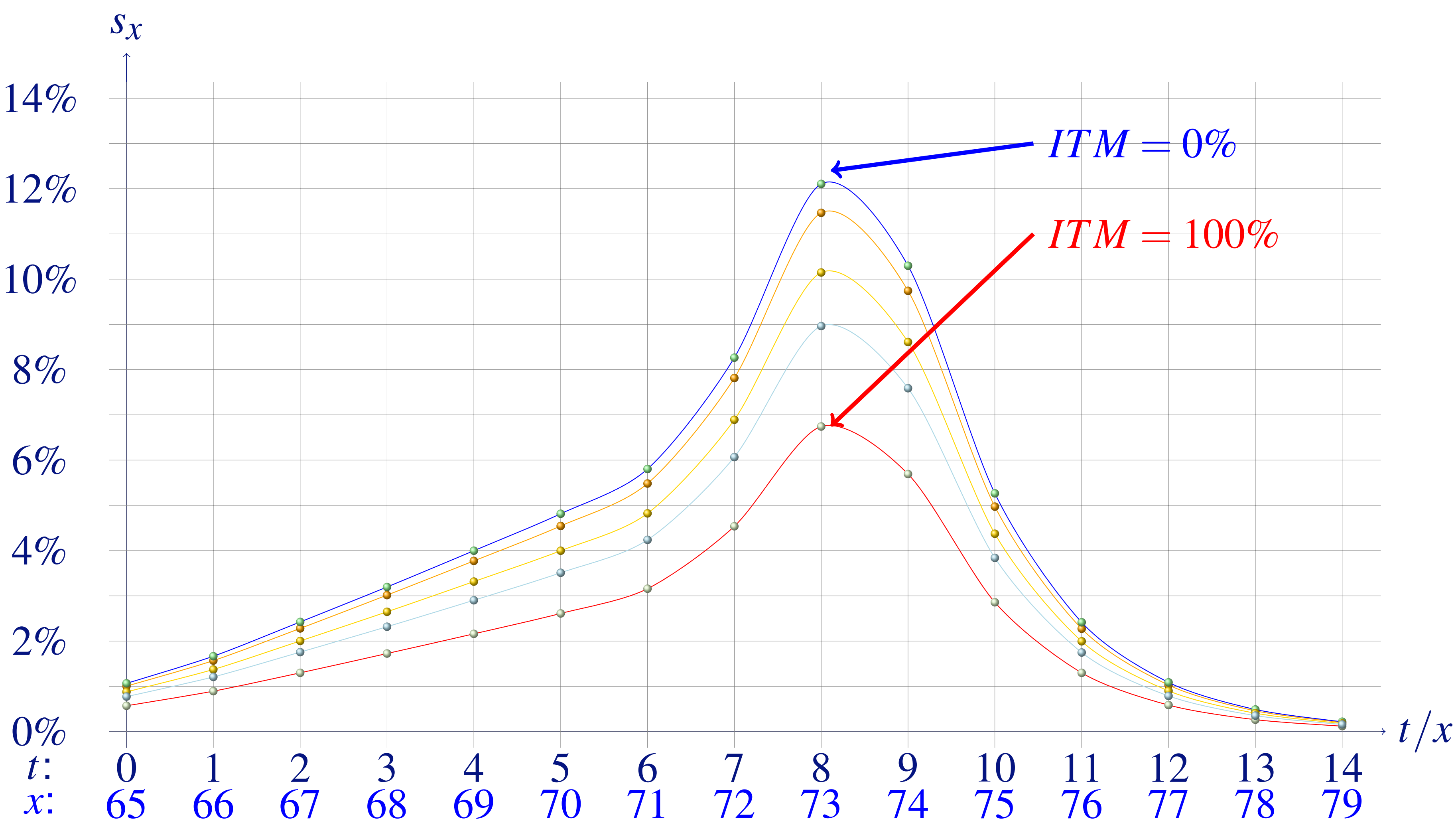
To illustrate how the value of the hedge liability depends on lapse assumptions, assume that the best estimate lapses ("BE") indicated above (eg 4% for all years, except for year 10 where lapses are 12%) were inaccurate and need to be revalued to 8% at year 10 and 2% thereafter ("New BE"). The following table shows the value of the valuation portfolio as at time 2. Note that maturity is now in 23 years.

	Instrument	Value Normal	Value BE	Value New BE	Value P&L
1	Put Fund	25.6	23.5	23.5	–
2	Put Fund	39.7	35.1	35.1	–
3	Put Fund	55.7	47.3	47.3	–
...					
7	Put Fund	156.7	112.6	112.6	–
8	Put Fund	195.4	134.6	134.6	–
9	Put Fund	241.2	149.3	149.3	0.0
...					
23	Put Fund	36986.0	10138.3	14802.2	-4663.9
	Total	42844.5	12973.2	18006.3	**-5033.1**

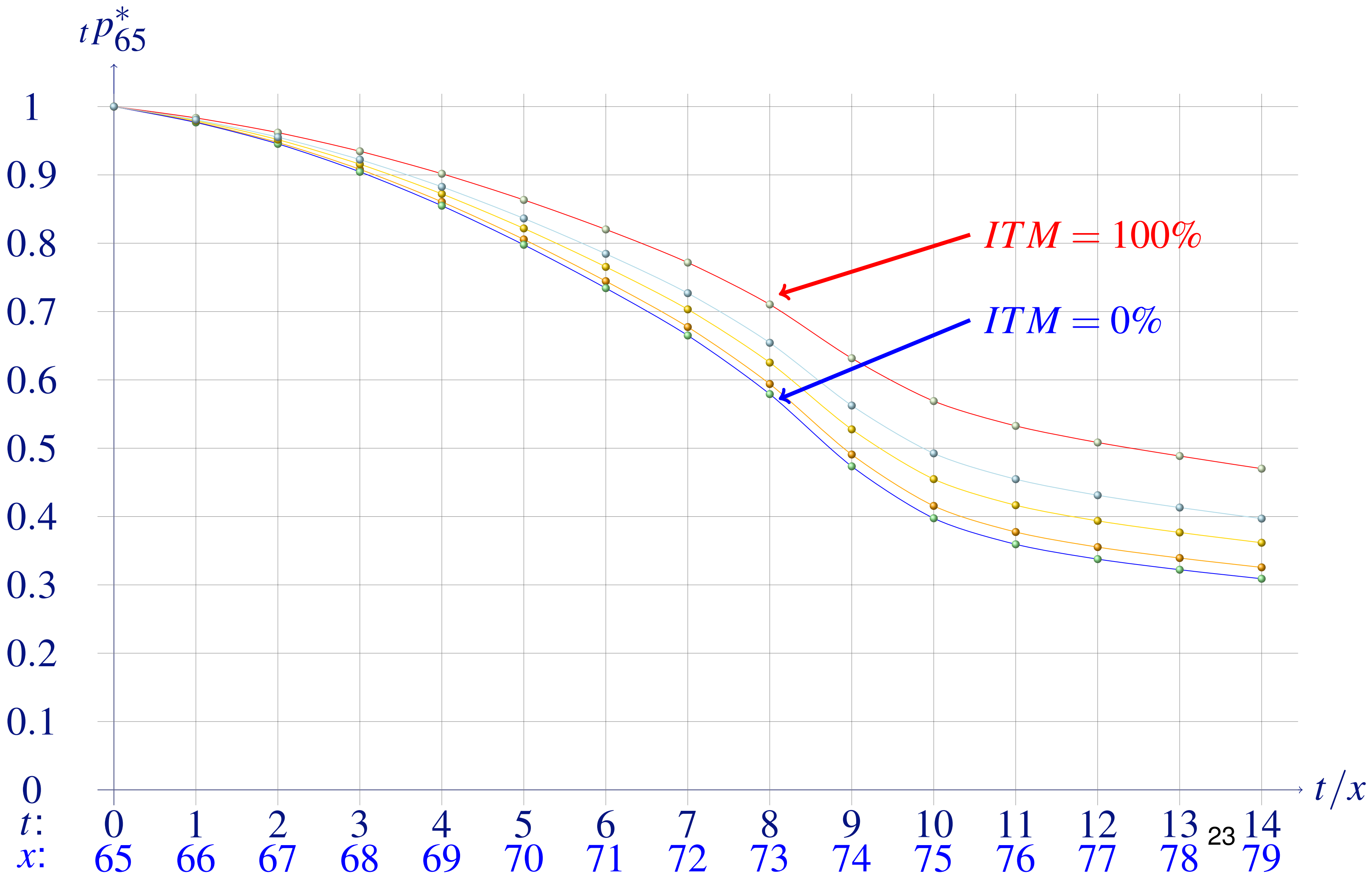
Lapse depend on moneyness (1/3)

$$
\begin{aligned}
\pi(GMWB) &= E^Q\left[\sum_{k\in\mathbb{N}} v^k \max\{0,(R_t - FV_t)\} \times I_\star(k)\right] \\
&= \sum_{k\in\mathbb{N}} v^k E^Q\left[\max\{0,(R_t - FV_t)\} \times I_\star(k)\right] \\
&= \sum_{k\in\mathbb{N}} v^k E^Q\left[E^Q\left[\max\{0,(R_t - FV_t)\} \times I_\star(k)|\mathcal{G}_k\right]\right] \\
&= \sum_{k\in\mathbb{N}} v^k E^Q\left[\max\{0,(R_t - FV_t)\} \times E^Q\left[I_\star(k)|\mathcal{G}_k\right]\right].
\end{aligned}
$$

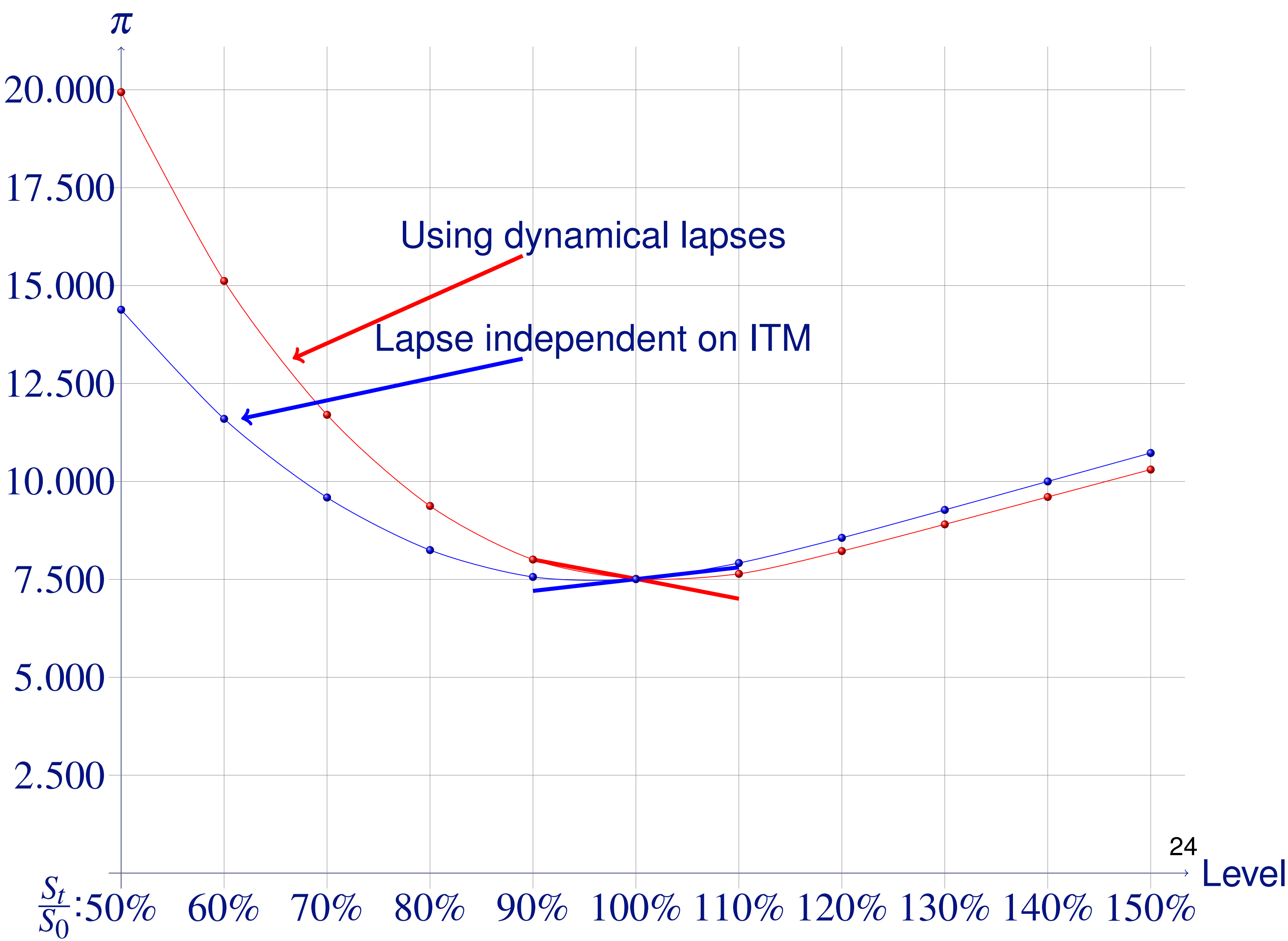
Lapse depend on moneyness (2/3)



Lapse depend on moneyness (3/3)



... and so does the response function



Price of a GMWB guarantee - Product definition (1/3)

$FV(t)$	denotes fund value at time t (before withdrawal),
$GV(t)$	denotes benefit base (guaranteed) value at time t,
$R(t)$	annuity paid at time t
$\psi(t, t+\Delta t)$	denotes the fund performance from time t to $t+\Delta t$, and
$\mathfrak{T} \subset \mathbb{R}^{+}$	denotes the set of times at which a ratchet takes place.

Price of a GMWB guarantee - Product definition (2/3)

In this example we have the following (assuming for sake of simplicity that $\mathfrak{T} \subset \{k \times \Delta t | k \in \mathbb{N}_0\}$ and also that annuity payments only take place at direct times $k\Delta t$ for some $k \in \mathbb{N}_0$):

$$
\begin{aligned}
FV(0) &= EE > 0 \\
GV(0) &= FV(0) \\
FV((k+1)\Delta t) &= (FV(k\Delta t) - R(k\Delta t))\,\psi(k\Delta t, (k+1)\Delta t) \\
GV((k+1)\Delta t) &= \begin{cases} \max(GV(k\Delta t), FV((k+1)\Delta t) - R((k+1)\Delta t)) & \text{if } (k+1)\Delta t \in \mathfrak{T}, \\ GV(k\Delta t) & \text{else.} \end{cases}
\end{aligned}
$$

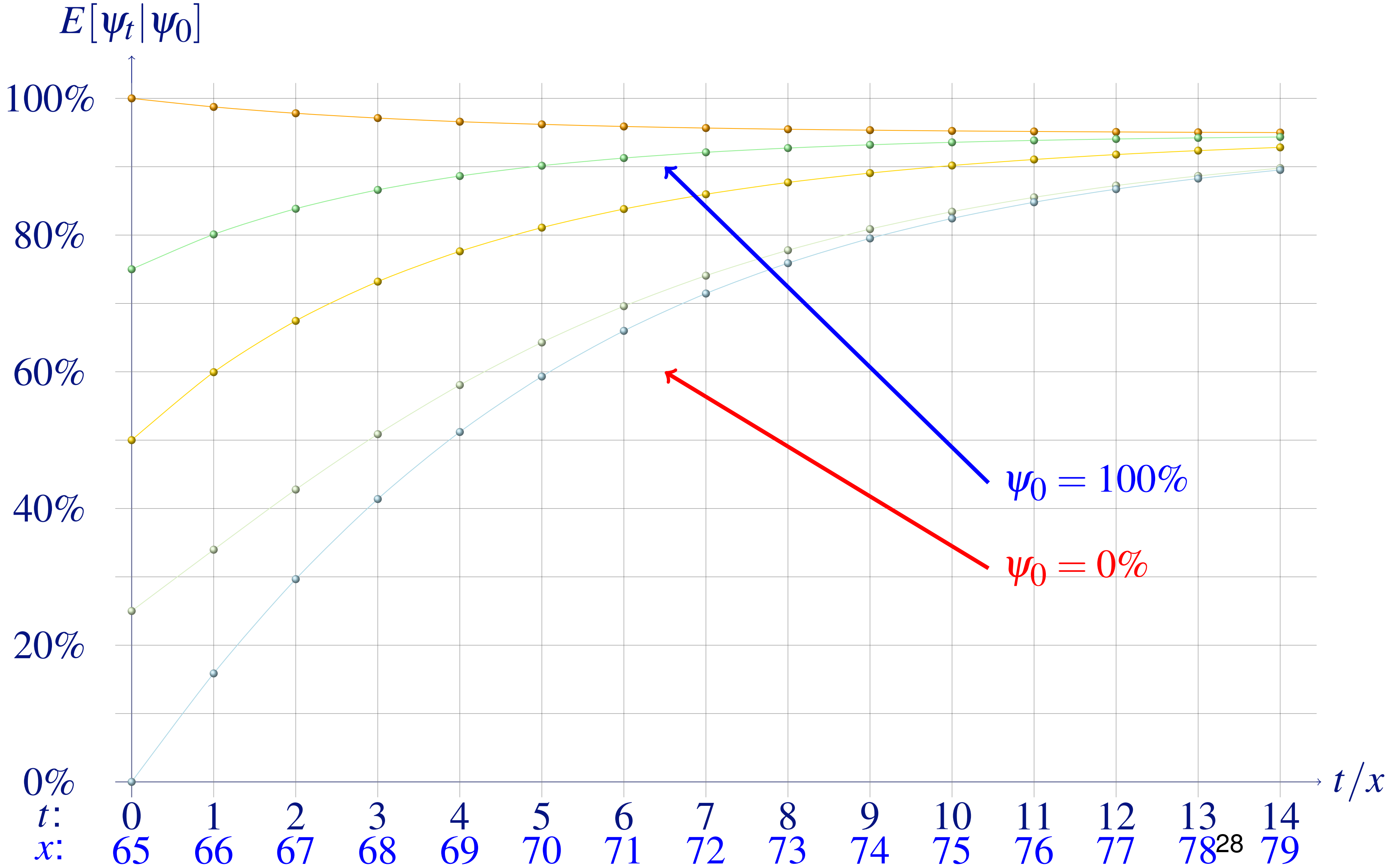
The death benefit is defined as the maximum of the current fund's value and the difference between the current $GV(t)$ and the annuities paid out before this point (eg $\sum_{k\in\mathbb{N}_0}^{k\Delta t \leq t} R(k\Delta t)$ until age 85. Afterwards there is no death benefit.

Price of a GMWB guarantee - Product definition (3/3)

The annuity can be withdrawn at times $\mathfrak{S} \subset \{k \times \Delta t | k \in \mathbb{N}_0\}$ and it amounts at time $t \in \mathfrak{S}$ to $\rho(\xi_0) \times GV(t) \times I_\star(t) \times \psi(t)$, ξ_0 is the first time $\xi_0 \in \mathfrak{S}$ where the person can withdraw. The person is allowed to withdraw less than this amount in line with the model as defined beforehand.

We assume a 65 year old policyholder who invests 100'000 $.

Utilisation - what is this?



Utilisation Model

Finally we look at an abstract example of the above concept. We assume for the sake of simplicity a time-homogeneous Markov chain and we consider a $x = 65$ year old man. To model the transition matrix $P(1)$ we assume the following:

	100%	75 %	50%	25%	0%
100%	0.95	0.5	–	–	–
75%	$1-\alpha$	0.95α	0.05α	–	–
50%	$0.8\,(1-\alpha)$	$0.2\,(1-\alpha)$	0.95α	0.05α	–
25%	$0.1\,(1-\beta)$	$0.1\,(1-\beta)$	$0.8\,(1-\beta)$	β	–
0%	$0.1\,(1-\beta)$	$0.1\,(1-\beta)$	$0.8\,(1-\beta)$	–	β

with

$$\alpha = \sqrt[5]{1-75\%}$$
$$\beta = \sqrt[5]{1-80\%}$$

Modified Model including Utilisation (1/2)

$$\begin{aligned}\pi(GMWB|\psi_o = i) &= E^Q\left[\sum_{k\in\mathbb{N}} v^k \max\{0,(R_t\times\psi_t - FV_t)\}\times I_\star(k)|\psi_o = i\right]\\ &= \sum_{k\in\mathbb{N}} v^k E^Q\left[E^Q\left[\max\{0,(R_t\times\psi_t - FV_t)\}\times I_\star(k)|\mathscr{G}_k\right]|\psi_o = i\right].\end{aligned}$$

Since we have assumed stochastic independence of ψ from the capital market variables, we need to first calculate

$$\begin{aligned}E^Q\left[\psi_t|\mathscr{G}_0\right] &= E^Q\left[\psi_t|\,\psi_0\right]\\ &= \sum_{j\in S} j\times p_{ij}(0,t).\end{aligned}$$

Modified Model including Utilisation (2/2)

Moreover if we assume that ψ_t and $I_\star(t)$ are independent, we can calculate $\pi(GMWB)$ as follows:

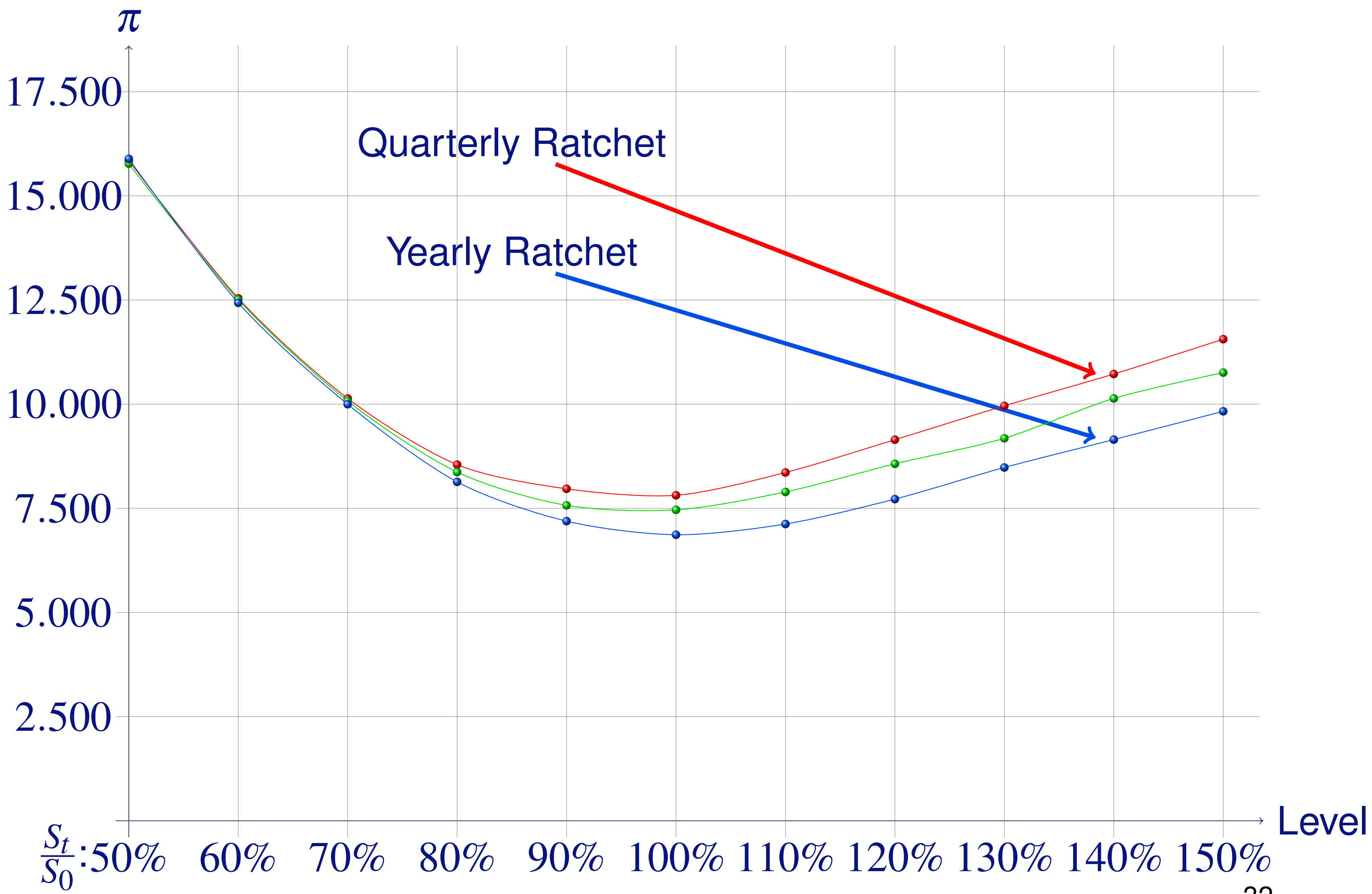
$$\pi(GMWB|\psi_o = i) = E^Q\left[\sum_{k\in\mathbb{N}} v^k \max\{0,(R_t\times\psi_t - FV_t)\}\times I_\star(k)\Big|\psi_o = i\right]$$

$$= \sum_{k\in\mathbb{N}} v^k E^Q\left[E^Q[\max\{0,(R_t\times\psi_t - FV_t)\}\times I_\star(k)|\mathscr{G}_k]\,|\psi_o = i\right]$$

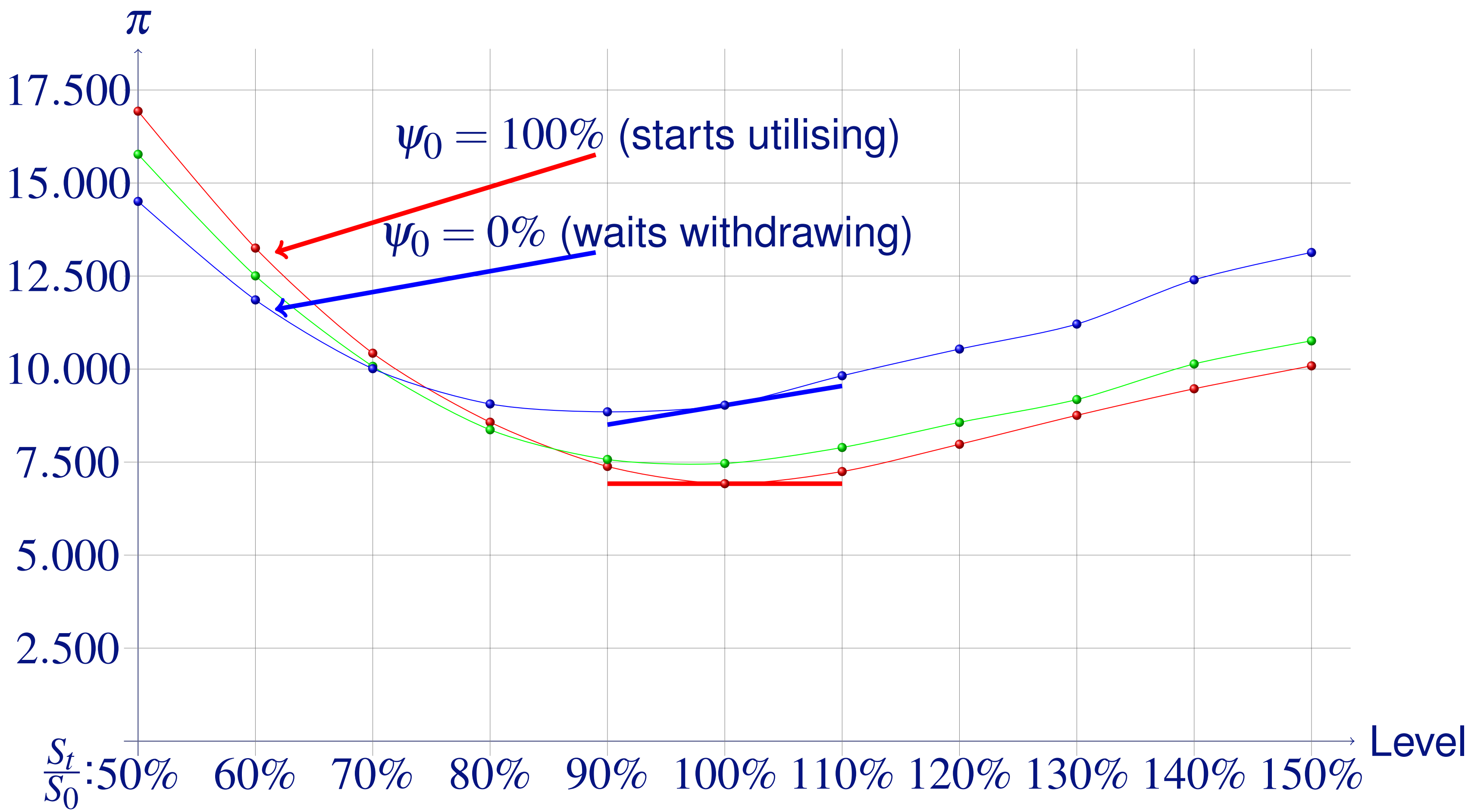
$$= \sum_{k\in\mathbb{N}} v^k E^Q\left[\max\{0,(R_t\times E[\psi_t|\psi_o = i] - FV_t)\}\times E^Q[I_\star(k)|\mathscr{G}_k]\right]$$

$$= \sum_{k\in\mathbb{N}} v^k E^Q[\max\{0,(R_t\times\{\sum_{j\in S} j\times p_{ij}(0,t)\} - FV_t)\}\times E^Q[I_\star(k)|\mathscr{G}_k]].$$

Utilisation example (1/2)



Utilisation example (2/2)



Hedging depends on PH behaviour - Model risk!

Hedging different effects

Easier to hedge

- Short dated options,
- Equity prices,
- Short term volatility,
- Interest rates.

More difficult to hedge

- Long dated options,
- Long term volatility,
- Long term interest rates,
- Policyholder behaviour (lapses, . . .),
- Basis risk.

Things to consider for VA risk management from a Board perspective

The following dimensions need to be considered:

Shortfall Risk: What is the intrinsic shortfall risk for the VA protfolio with respect to the various metrics?

Product Risk: What are the product risks within the portfolio and how are they managed?

Hedging Risk: How does the hedging strategy address the risks and what are the risks induced by the hedging strategy?

Clarity is also needed regarding risk appetite and the hedging strategy.

Good Practices

1. Define risk limits and take them seriously.
2. Do not underestimate the benefits of diversification (in one's business model).
3. Carry out scenario analyses and stress tests.
4. Monitor traders carefully.
5. Do not blindly trust models.
6. Do not sell clients inappropriate products.
7. Do not ignore liquidity risk.
8. Do not finance long-term assets with short-term liabilities.
9. Make sure a hedger does not become a speculator.

Q&A